Symmetric Relaying Strategy for Two-Relay Networks

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***Abstract*—In this paper, we propose a concept of relaying named symetric relaying strategy for two relay network. In our strategy, we have two relays in the network and both relays can completely decode the message transmitted by the other relay. The proposed rate is shown to subsume the previously proposed rate for feed-forward relay network based on decode-and-forward.**

***Index Terms*—Relay network, achievable rate, symmetric relaying, partial decode-and-forward.**

I. Introduction

The discrete-memoryless relay network denoted by $(\mathcal{X}_0 \times \mathcal{X}_1 \times ... \times \mathcal{X}_N, p(y_0, y_1, ..., y_N|x_0, x_1, ..., x_N), \mathcal{Y}_0 \times \mathcal{Y}_1 \times ... \times \mathcal{Y}_N)$ consists of a sender $X_0 \in \mathcal{X}_0$, a receiver $Y_0 \in \mathcal{Y}_0$, relay senders $X_1 \in \mathcal{X}_1$,...,$X_N \in \mathcal{X}_N$ and relay receivers $Y_1 \in \mathcal{Y}_1$,...,$Y_N \in \mathcal{Y}_N$ and a family of conditional probability mass functions $p(y_0, y_1, ..., y_N|x_0, x_1, ..., x_N)$ on $\mathcal{Y}_0 \times \mathcal{Y}_1 \times ... \times \mathcal{Y}_N$ one for each $(x_0, x_1, ..., x_N) \in \mathcal{X}_0 \times \mathcal{X}_1 \times ... \times \mathcal{X}_N$. A $(2^{nR}, n)$ code for the channel consists of: i) a set of messages $1, 2, ..., 2^{nR}$, ii) an encoding function that maps each message w into a codeword $x^n(w)$ of length n, iii) relay encoding functions $x_{1i} = f_i(y_{11}, y_{12}, ..., y_{1,i-1})$, for $1 \leq i \leq n$, and iv) a decoding function that maps each received sequence y^n into an estimate $\hat{w}(y^n)$. A rate R is achievable if there exists a sequence of $(2^{nR}, n)$ codes with $P_e^{(n)} = P(\hat{W} \neq W) \to 0$, as $n \to \infty$. Channel capacity $\mathcal{C}$ is defined as the supremum over the set of achievable rates.

There are two common protocols for relaying in a network: 1) decode-and-forward and 2) compress-and-forward proposed in [1], that are extensively used for relaying in the networks [3]-[8]. In the previously proposed relaying methods specially those which are based on decode-and-forward [3]-[6], it is assumed that the relays are arranged in the feed-forward structure from the source to the destination, i.e. the message transmitted by the ith relay can only be decoded by the jth relay $(j > i)$, and cannot be decoded by the previous relays. In [8], a relaying strategy for two relay network was proposed, where each relay can partially decode the message of seder and the other relay, it was named parallel relaying strategy in contrast with the sequential relaying concept.

In this paper, we propose a simplified parallel relaying strategy for two relay network. In contrast with the previously proposed methods [8], in our proposed method, each relay can completely decode the message transmitted by the other relay in addition to the part of the message transmitted by the source. However the proposed method can be considered as a special case of more complicated form proposed in [8], but it yields a better and simplified understanding of parallel relaying strategy. And it can be considered as a primary text of this concept. Moreover, we change the name of parallel relaying method to symmetric relaying method because in some literature [10]-[11], parallel relay network is refered to the network in that the relays don't interchange any information.

II. Symmetric Relaying Scheme

In [2], partial decoding is defined as a special case of Theorem 7 in [1]. In this scheme, the relay does not completely decode the transmitted message by the sender. Instead the relay only decodes part of the message transmitted by the sender. Here, we apply the concept of partial decoding scheme to the relay networks with two relays and present a new achievable based on a symmetric relaying strategy in the following theorem.

Theorem 1: For any relay networks $(\mathcal{X}_0 \times \mathcal{X}_1 \times \mathcal{X}_2, p(y_0, y_1, y_2|x_0, x_1, x_2), \mathcal{Y}_0 \times \mathcal{Y}_1 \times \mathcal{Y}_2)$ the capacity $\mathcal{C}$ is lower bounded by

$$\begin{aligned} \mathcal{C} \geq \sup_{p(v,u_{01},u_{02},x_0,x_1,x_2)} &\min\{I(X_0X_1X_2;Y_0), \\ &I(U_{01};Y_1|X_2X_1V) + I(U_{02}X_2;Y_1|X_1V) \\ &\quad +I(X_0;Y_0|X_1X_2U_{01}U_{02}V), \\ &I(U_{02};Y_2|X_1X_2V) + I(U_{01}X_1;Y_2|X_2V) \\ &\quad +I(X_0;Y_0|X_1X_2U_{01}U_{02}V), \\ &I(U_{02}X_2;Y_1|X_1V) + I(U_{01}X_1;Y_2|X_2V) \\ &\quad +I(X_0;Y_0|X_1X_2U_{01}U_{02}V), \\ &I(U_{01};Y_1|X_1X_2V) + I(U_{02};Y_2|X_1X_2V) \\ &-I(U_{01};U_{02}|X_1X_2V) + I(X_0;Y_0|X_1X_2U_{01}U_{02}V)\} \end{aligned} \tag{1}$$

where the supremum is over all joint probability mass function $p(x_0, u_{01}, u_{02}, x_1, x_2, v)$ on the product set, $\mathcal{V} \times \mathcal{U}_{01} \times \mathcal{U}_{02} \times \mathcal{X}_0 \times \mathcal{X}_1 \times \mathcal{X}_2$, such that

$$(VU_{01}U_{02}) \to (X_0X_1X_2) \to Y_0 \tag{2}$$

Proof: The message transmitted by the source is divided into three independent parts. The first and second parts are decoded directly by the first and second relays respectively and the receiver can only make estimates of them, while the third part is directly decoded by the receiver. Also the second part of the message can be decoded by the first relay after it is decoded by the second relay and the first part of the message can be decoded by the second relay after it is decoded by the first relay. The sender and the relays cooperate in the next transmission blocks to remove the receiver's uncertainly about

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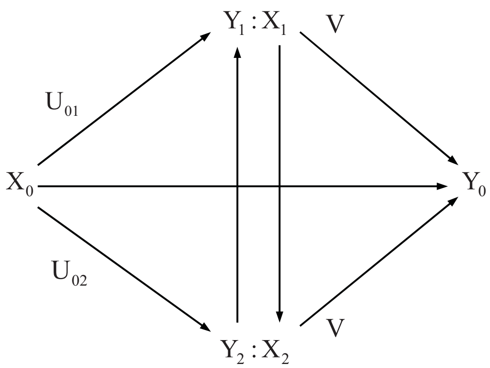

Fig. 1. Relay networks with the individual parts of the messages at the sender and the relays shown in it.

the first and second parts of the message. Fig. 1 shows the individual parts of the messages at the sender and the relays in the proposed coding scheme.

Consider a block Markov encoding scheme with B blocks of transmission, each of n symbols. A sequence of $(B-2)$ messages w_i, $i=1,2,...,B-2$, each selected independently and uniformly over $\mathcal{W}$ is to be sent in nB transmissions. (Note that as $B\to\infty$, for fixed n, the rate $R(B-2)/B$ is arbitrarily close to R). The same codebook is used in each block of transmission. We show that for any joint probability mass function $p(x_0,x_1,x_2,u_{01},u_{02},v)$ there exists at least a sequence of codebooks $\mathcal{C}_n$, such that $P_e^{(n)}\to 0$ as $n\to\infty$ if $R\le C$, where

$$R = R_{00}+R_{01}+R_{02} \tag{3}$$

Random Coding:

For any joint probability mass function

$$\begin{aligned} &p(x_0,u_{01},u_{02},x_1,x_2,v)\\ &=p(x_0|u_{01},u_{02},x_1,x_2,v)p(u_{01}|x_1,x_2,v)\\ &\quad p(u_{02}|x_1,x_2,v)p(x_1|v)p(x_2|v)p(v)\end{aligned} \tag{4}$$

on the product set

$$\mathcal{V}\times\mathcal{U}_{01}\times\mathcal{U}_{02}\times\mathcal{X}_0\times\mathcal{X}_1\times\mathcal{X}_2.$$

a) Generate $2^{n(R_{01}+R_{02})}$ i.i.d. v^n sequences each with probability $p(v^n)=\prod_{i=1}^{n}p(v_i)$. Index them as $v^n(m_v)$, where $m_v\in[1,2^{n(R_{01}+R_{02})}]$.
b) For each $v^n(m_v)$, generate $2^{nR_{01}}$ i.i.d. x_1^n sequences each with probability $p(x_1^n|v^n)=\prod_{i=1}^{n}p(x_{1i}|v_i)$. Index them as $x_1^n(m_1|m_v)$, where $m_1\in[1,2^{nR_{01}}]$.
c) For each $v^n(m_v)$, generate $2^{nR_{02}}$ i.i.d. x_2^n sequences each with probability $p(x_2^n|v^n)=\prod_{i=1}^{n}p(x_{2i}|v_i)$. Index them as $x_2^n(m_2|m_v)$, where $m_2\in[1,2^{nR_{02}}]$.
d) For each $v^n(m_v)$, $x_1^n(m_1|m_v)$ and $x_2^n(m_2|m_v)$, generate $2^{nR_{01}}$ i.i.d. u_{01}^n sequences each with probability $p(u_{01}^n|x_1^n,x_2^n,v^n)=\prod_{i=1}^{n}p(u_{01,i}|x_{1i},x_{2i},v_i)$. Index them as $u_{01}^n(m_{01}|m_1,m_2,m_v)$, where $m_{01}\in[1,2^{nR_{01}}]$.
e) For each $v^n(m_v)$, $x_1^n(m_1|m_v)$ and $x_2^n(m_2|m_v)$, generate $2^{nR_{02}}$ i.i.d. u_{02}^n sequences each with probability $p(u_{02}^n|x_1^n,x_2^n,v^n)=\prod_{i=1}^{n}p(u_{02,i}|x_{1i},x_{2i},v_i)$. Index them as $u_{02}^n(m_{02}|m_1,m_2,m_v)$, where $m_{02}\in[1,2^{nR_{02}}]$.
f) For each $v^n(m_v)$, $x_1^n(m_1|m_v)$, $x_2^n(m_2|m_v)$, $u_{01}^n(m_{01}|m_1,m_2,m_v)$ and $u_{02}^n(m_{02}|m_1,m_2,m_v)$, generate $2^{nR_{00}}$ i.i.d x_0^n sequences each with probability

$$\begin{aligned} &p(x_0^n|u_{01}^n,u_{02}^n,x_1^n,x_2^n,v^n)\\ &=\prod_{i=1}^{n}p(x_{0,i}|u_{01,i},u_{02,i},x_{1,i},x_{2,i},v_i)\end{aligned}$$

Index them as $x_0^n(m_{00,i}|m_{01,i},m_{02,i},m_{1,i},m_{2,i},m_{v,i})$, $m_{00}\in[1,2^{nR_{00}}]$.

The index m_1 represents the index m_{01} of the previous block. The index m_2 represents the index m_{02} of the previous block. The pair m_v is equivalent to the pair (m_1,m_2) of the previous block.

This defines the joint codebook $\mathcal{C}_0$ for all the transmitter nodes. Repeating the above process a)-f) independently, we generate another random codebooks C_1 similar to $\mathcal{C}_0$. We will use these 2 codebooks in a sequential way as follows: In block $b=1,...,B$, the codebook $\mathcal{C}_{b \text{ mode } 2}$ is used. Hence in any 2 consecutive blocks, codewords from different blocks are independent. This is a property we will use in the analysis of the probability of error.

Encoding: Encoding is performed in the following Markov fashion: Let $w_i=(m_{00,i},m_{01,i},m_{02,i})$ be the new message to be transmitted in block i. Assume that the first and second relays have estimates of $(\hat{\hat{m}}_{01,i-1},\hat{\hat{m}}_{2,i-1})$ and $(\hat{\hat{m}}_{02,i-1},\hat{\hat{m}}_{1,i-1})$ of the previous indices, sent by the sender and the other relay, respectively. In this fashion, the first and second relays transmissions in block i are $x_1^n(m_{1,i}|m_{v,i})$ and $x_2^n(m_{2,i}|m_{v,i})$ respectively, where $m_{1,i}=\hat{\hat{m}}_{01,i-1}$ and $m_{2,i}=\hat{\hat{m}}_{02,i-1}$. The sender at block i knowing $m_{01,i}$, $m_{02,i}$ (the parts of the messages transmitted to the relays in block i), $m_{01,i-1}$, $m_{02,i-1}$ and hence $m_{1,i}$, $m_{2,i}$, m_v, transmits $x_0^n(m_{00,i}|m_{01,i},m_{02,i},m_{1,i},m_{2,i},m_{v,i})$ implicity including $u_{01}^n(m_{01}|m_1,m_2,m_v)$, $u_{02}^n(m_{02}|m_1,m_2,m_v)$.

Decoding: Assume that at the end of block $(i-1)$, the first relay knows $(\hat{\hat{m}}_{01,1},\hat{\hat{m}}_{01,2},...,\hat{\hat{m}}_{01,i-1})$, $(\hat{\hat{m}}_{2,1},\hat{\hat{m}}_{2,2},...,\hat{\hat{m}}_{2,i-1})$, $(\hat{\hat{m}}_{1,1},\hat{\hat{m}}_{1,2},...,\hat{\hat{m}}_{1,i})$ and $(\hat{\hat{m}}_{v,1},\hat{\hat{m}}_{v,2},...,\hat{\hat{m}}_{v,i})$. The second relay knows $(\hat{\hat{m}}_{02,1},\hat{\hat{m}}_{02,2},...,\hat{\hat{m}}_{02,i-1})$, $(\hat{\hat{m}}_{1,1},\hat{\hat{m}}_{1,2},...,\hat{\hat{m}}_{1,i-1})$, $(\hat{\hat{m}}_{2,1},\hat{\hat{m}}_{2,2},...,\hat{\hat{m}}_{2,i})$ and $(\hat{\hat{m}}_{v,1},\hat{\hat{m}}_{v,2},...,\hat{\hat{m}}_{v,i})$.At the end of block i

1) (At the first relay) Knowing $\hat{\hat{m}}_{v,i-1}$, $\hat{\hat{m}}_{1,i-1}$, $\hat{\hat{m}}_{2,i-1}$, $\hat{\hat{m}}_{v,i}$, $\hat{\hat{m}}_{1,i}$, the first relay declares $\hat{\hat{m}}_{2,i}=m_{2,i}$ or equivalently $\hat{\hat{m}}_{02,i-1}=m_{02,i-1}$ were sent by looking for unique indices m_2 and m_{02} such that

$$\begin{gathered}\left(\ x_2^n(\hat{\hat{m}}_{2,i}|\hat{\hat{m}}_{v_i}),x_1^n(\hat{\hat{m}}_{1,i}|\hat{\hat{m}}_{v_i}),v^n(\hat{\hat{m}}_{v_i}),y_1^n(i)\ \right)\in A_\epsilon^n;\\ \begin{pmatrix} u_{02}^n(\hat{\hat{m}}_{02,i-1}|\hat{\hat{m}}_{1,i-1},\hat{\hat{m}}_{2,i-1},\hat{\hat{m}}_{v_{i-1}}),\\ x_2^n(\hat{\hat{m}}_{2,i-1}|\hat{\hat{m}}_{v_{i-1}}),x_1^n(\hat{\hat{m}}_{1,i-1}|\hat{\hat{m}}_{v_{i-1}}),\\ v^n(\hat{\hat{m}}_{v_{i-1}}),y_1^n(i-1)\end{pmatrix}\in A_\epsilon^n;\end{gathered} \tag{5}$$

this can be done with arbitrary small probability of error if n is sufficiently large and

$$R_{02} \quad < \quad I(U_{02}X_2;Y_1|X_1V) \tag{6}$$

The proof can be done based on regular encoding/sliding window decoding method [3] by respect to the fact that the codewords of two consecutive block are independent.

2) (At the first relay) Knowing $\hat{m}_{v,i}$, $\hat{\hat{m}}_{1,i}$, $\hat{\hat{m}}_{2,i}$, the first relay declares $\hat{\hat{m}}_{01,i} = m_{01,i}$ was sent by looking for unique index m_2 such that

$$\left(\begin{array}{c} u_{01}^n(\hat{\hat{m}}_{01,i}|\hat{\hat{m}}_{1,i}, \hat{\hat{m}}_{2,i}, \hat{m}_{v_i}), x_2^n(\hat{m}_{2,i}|\hat{\hat{m}}_{v_i}), \\ x_1^n(\hat{m}_{1,i}|\hat{\hat{m}}_{v_i}), v^n(\hat{\hat{m}}_{v_i}), y_1^n(i) \end{array} \right) \in A_\epsilon^n;$$

this can be done with arbitrary small probability of error if n is sufficiently large and

$$R_{01} < I(U_{01}; Y_1|X_1X_2V) \tag{7}$$

3) (At the second relay) Due to the symmetry in the definition of the codewords by the same argument as previous, the following rate region is obtained for the second relay,

$$\begin{aligned} R_{01} &< I(U_{01}X_1; Y_2|X_2V) & (8) \\ R_{02} &< I(U_{02}; Y_2|X_1X_2V) & (9) \end{aligned}$$

4) (Jointly typical $(u_{01}^n,\ u_{02}^n)$) For each product bin $[1, 2^{nR_{01}}] \times [1, 2^{nR_{02}}]$, jointly typical pair (u_{01}^n, u_{02}^n) can be found if

$$\begin{aligned} R_{01} + R_{02} < \quad & I(U_{01}; Y_1|X_1X_2V) + I(U_{02}; Y_2|X_1X_2V) \\ & -I(U_{01}; U_{02}|X_1X_2V) \end{aligned} \tag{10}$$

The proof can be done based on product binning method as mentioned for the proof of Marton's rate region for broadcast channel in [9].

5) (Backward decoding at the destination) Assume that $\hat{m}_{1,i}$, $\hat{m}_{2,i}$, $\hat{m}_{01,i-1}$ and $\hat{m}_{02,i-1}$ have been decoded accurately. The sink determines $\hat{m}_{01,i-2}$ and $\hat{m}_{02,i-2}$ were sent such that

$$\left(\begin{array}{c} u_{01}^n(\hat{m}_{01,i}|\hat{m}_{1,i}, \hat{m}_{2,i}, \hat{m}_{v,i}), x_1^n(\hat{m}_{2,i}|\hat{m}_{v,i}), \\ u_{02}^n(\hat{m}_{02,i}|\hat{m}_{1,i}, \hat{m}_{2,i}, \hat{m}_{v,i}), \\ x_2^n(\hat{m}_{1,i}|\hat{m}_{v,i}), v^n(\hat{m}_{v_i}), y_0^n(i) \end{array} \right) \in A_\epsilon^n$$

where $\hat{m}_{1,i} = \hat{m}_{01,i-1}$, $\hat{m}_{2,i} = \hat{m}_{02,i-1}$ and $\hat{m}_{v,i} \triangleq (\hat{m}_{01,i-2}, \hat{m}_{02,i-2})$. This can be done with arbitrary small probability of error if n is sufficiently large and

$$R_{01} + R_{02} < I(U_{01}U_{02}X_1X_2V; Y_0). \tag{11}$$

It should be noted that without definition of V, we should confine R_{01} and R_{02}, to $R_{01} < I(X_1; Y_0|X_2)$, $R_{02} < I(X_2; Y_0|X_1)$, but by introducing V, these conditions are removed and (11) is obtained.

6) (At the destination) Knowing By knowing $\hat{m}_{v,i}$, $\hat{m}_{1,i}$, $\hat{m}_{2,i}$, $\hat{m}_{01,i}$, $\hat{m}_{02,i}$, the sink declares $\hat{m}_{00,i} = m_{00,i}$ is sent if it is a unique index $m_{00,i}$ such that

$$\left(\begin{array}{c} x_0^n(\hat{m}_{00,i}|\hat{m}_{01,i}, \hat{m}_{02,i}, \hat{m}_{1,i}, \hat{m}_{2,i}, \hat{m}_{v,i}), \\ u_{01}^n(\hat{m}_{01,i}|\hat{m}_{1,i}, \hat{m}_{2,i}, \hat{m}_{v,i}), \\ u_{02}^n(\hat{m}_{02,i}|\hat{m}_{1,i}, \hat{m}_{2,i}, \hat{m}_{v,i}), x_1^n(\hat{m}_{1,i}|\hat{m}_{v,i}), \\ x_2^n(\hat{m}_{2,i}|\hat{m}_{v,i}), v^n(\hat{m}_{v_i}), y_0^n(i) \end{array} \right) \in A_\epsilon^n$$

This can be done with arbitrary small probability of error if n is sufficiently large and

$$R_{00} < I(X_0; Y_0|U_{01}U_{02}X_1X_2V) \tag{12}$$

From the above constraints, we obtain the following results: (2), (3), (11) and (12) results the first term of (1). (2), (3), (6), (7) and (12) results the second term of (1). (2), (3), (8), (9) and (12) results the third term of (1). (2), (3), (8), (6), and (12) results the fourth term of (1). (3), (10) and (12) results the fifth term of (1).

This completes the proof of Theorem 1. □

Remark:

1) By substituting $U_{01} = X_0$, $U_{02} = 0$ and $V = X_2$ in (1), the best proposed rate until now based on decode-and-forward [3], is obtained as follows,

$$\begin{aligned} \mathcal{C} \geq \sup_{p(x_0,x_1,x_2)} \min \quad & \{I(X_0X_1X_2; Y_0), I(X_0X_1; Y_2|X_2), \\ & I(X_0; Y_1|X_1X_2)\} \end{aligned} \tag{13}$$

III. Conclusion

In this paper, by taking advantage of joint three new techniques 1) product binnig [9], 2) regular encoding/sliding window decoding [3], 3) regular encoding/backward decoding [8], a new concept of relaying named symmetric relaying strategy is introduced and is shown to subsume the well-known proposed rate for the feed-forward relay network based on decode-and-forward [3].

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