Compress-and-Forward Strategy for the Relay Channel with Causal State Information

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Abstract—In this paper, we study a state-dependent relay channel where perfect Channel State Information (CSI) is causally known to transmitters in both symmetric and asymmetric manner. So, three different setups are investigated in which causal CSI is available: 1) at the relay only, 2) at the source only and 3) both at the source and the relay nodes. In each situation, we obtain the lower bound on the capacity (achievable rate) for the general discrete memoryless case. The lower bounds are derived based on Shannon's strategy where CSI is known, and Compress-and-Forward (CF) Strategy at the relay.

Index Terms—relay channel, Compress-and-Forward, causal channel state information.

I. INTRODUCTION

Because of the wide range of applications, the channels whose conditional output probability distribution is controlled by random parameters, and where the Channel State Information (CSI) is known at some nodes, have been widely studied over the years. Recently multiple user channels of this type have received great attention.

When CSI is available at the transmitter(s), two main situations could be distinguished, where transmitter knows the CSI causally or non-causally. In the causal case, the transmission, at every time instant, depends only on the past and present CSI, whereas in the non-causal case, the transmitter knows in advance the realization of the entire state sequence from the beginning to the end of the block. When the receiver is equipped with CSI, there is no difference between two cases, because it could wait until the end of the block before decoding. Also, Channel State Information at Receiver (CSIR) could be estimated via received signals. But, availability of Channel State Information at Transmitter (CSIT) requires feedback link to each of the transmitters. So, asymmetric CSIT would be common in practical problems.

Single-user channel model with CSI known causally at the transmitter, was introduced in 1958 by Shannon [1]. Shannon found the capacity of this channel, by showing that, this capacity is equal to the capacity of an ordinary discrete memoryless channel (DMC), with the same output alphabet and an extended input alphabet of size \(|X|^{|S|}\) (which consists of all mappings from \(S\) to \(X\)), where \(S\) and \(X\) indicate states and input alphabet, respectively. Caire and Shamai [2] studied the capacity of single user channel with causal partial CSIT and partial CSIR, both for independent and identically distributed (i.i.d) states and states with memory.

In [3], non-causal CSIT channel model was introduced in the context of coding for computer memories with defective cells, where non-causal CSIT is locations of defective cells which is known to the encoder non-causally. In 1980, capacity of this model was developed by Gel’fand and Pinsker [5] using a binning technique which is called Gel’fand-Pinsker (GP) coding. As it has been verified to be suitable for modeling in various problems, such as: intersymbol interference (ISI) channel, digital watermarking, and various broadcasting schemes, the power constrained additive Gaussian channel with additive interference is one of the most interesting example for non-causal CSIT [4], where additive interference serves as a channel state and is known at the transmitter. Choosing a specific auxiliary random variable for GP coding, Costa [6] found the capacity of this channel. His technique is referred to as dirty paper coding (DPC). He showed that, DPC achieves capacity of additive Gaussian channel without additive interference, which means that the effect of the additive channel state on the capacity is eliminated.

In spite of growing interest in state-dependent multi-user models, combating the effect of CSI is much more complicated in these channels. Because, one of the most important issues in this case, is the availability of CSI at all or only some of the users. Different state-dependent multi-user channels were already studied [4], [7]-[11]. Recently, besides the broadcast and the multiple-access channel, state-dependent relay channel has been also considered in some works [8], [9], [13] - [16]. Similar to the Costa’s result, it is shown in [9] that for the degraded Gaussian relay channel, DPC eliminates the effect of the additive channel state on the capacity (added to both relay’s input signal and receiver’s input signal), if symmetric full CSI is available (both at the source and the relay). Also, in [8] the capacity of the degraded state-dependent discrete memoryless relay channel has been derived, where the source and relay have identical causal CSI. The capacity achieving strategy in [8], consists of the extension of Shannon’s result [1] on the capacity of single user channels with state information and Decode-and-Forward (DF) scheme [12, Theorem 1], at the relay. Causal CSI problem is investigated for some classes of state-dependent relay channel in [13], where source, relay and destination know the partial causal CSI. The authors classified
this scenario into 4 distinct types (i.e., instantaneous, casual, delayless and relaying with unlimited look-ahead). Then, using Shannon’s strategy and DF coding, lower and upper bounds were obtained for theses channels.

In [14], capacity expressions for degraded discrete memoryless relay channels were obtained, where both causal and non-causal cases were considered and different CSI was known to the source and relay. Asymmetric CSI in the relay channel is considered in [15] and [16], which is known to the source or to the relay only in a non-causal manner, respectively, and lower and upper bounds on the capacity were derived. In both works, relay uses DF strategy and GP coding is used where CSI is available.

DF or partially DF (PDF) strategy is based on full or partial cooperation between the source and the relay, so the source must know the relay’s input [12], [17]-[19]. But, when only the relay knows the CSI, the source could not conceive what the relay exactly sends. In [16] to ameliorate this problem, codeword splitting is used, in which the relay input is split into two independent parts. This splitting prevents to comply the cooperation and DF strategy that we are looking for. As a confirmation, the results in [16] show that, even in degraded Gaussian channel, lower bound based on DF strategy in not tight. In this paper, in order to overcome this difficulty, we propose a coding scheme at the relay which is based on the CF strategy ([12, Theorem 6]) and where CSI is available at the relay, it is combined with Shannon’s strategy. The scheme is useful in this case, because independent codebooks are used at the source and the relay. Hence, it makes CF strategy, a suitable choice when asymmetric CSI is available at the nodes.

To further motivate our approach, we explain the cases where each strategy could help. DF (or PDF) coding schemes are based on decoding all (or part) of the message by the relay and cooperation of the source and the relay to send the decoded part to the receiver. In the cases that the channel from the source to the relay is remarkably better than the direct link between the source and the receiver, DF strategy could be beneficial. But, when these two links are similar in average or the direct link is better, another scheme is needed and DF could not be helpful, since the receiver could comprehend all the codewords which could be decoded by the relay [20]. In the latter case, another technique will be used that does not require the relay to decode any part of the transmitted message. In this technique, called as CF strategy, the relay compresses the received sequence and re-encodes and sends it to the receiver [21]. In the Gaussian relay channel, it was shown in [20] that CF outperforms DF considerably in the second scenario. Hence, in the relay channel controlled with random parameters, analogous to the classic case, we believe that preferable strategy is CF, when the link between the source and the relay is worse than the direct link. Recently, authors in [11] have shown for the general Gaussian relay channel with non-causal CSI, the cases in which the CF based lower bounds outperform the DF based lower bounds and can achieve the rates nearly close to the upper bounds.

In this paper, concentrating on CF strategy, we study three distinct scenarios, where CSI is available at the relay only, at the source only, both at the relay and the source, in a causal manner. So, asymmetric cases also are analyzed in this work. This channel model fits to the wireless networks, where some of the nodes can estimate the states of the channel, e.g. fading coefficients, in a causal manner and with high accuracy. This setup may model the basic building block for node cooperation over wireless networks in which some of the terminals may be equipped with cognition capabilities that permits them to estimate the channel states, e.g. receiver estimates the instantaneous state of the channel and sends it back to the transmitter via a feedback link. [4]. In all cases, we obtain lower bounds on the capacity using the CF coding scheme and Shannon’s strategy [2].

The rest of the paper is organized as follows. Section II introduces the state-dependent relay channel and the notations. In section III, we consider three different scenarios and derive lower bounds on the capacity for each case. Finally, section IV concludes the paper.

II. Preliminaries and Definitions

We use the following notations throughout the paper: upper case letters (e.g., \( X, Y \)), are used to denote random variables, and their realization are shown with lower case letters (e.g., \( x, y \)). \( X, Y, \ldots \) are used to designate alphabet sets. The probability mass function (p.m.f) of a random variable \( X \) is denoted by \( p_X(x) \), where occasionally subscript \( X \) is omitted. \( |\mathcal{X}| \) denotes the cardinality of a finite discrete set \( \mathcal{X} \). \( \mathcal{X}^n \) specifies the set of \( \epsilon \)-strongly, jointly typical sequences of length \( n \), on \( p(x,y) \). The notation \( X^n_j \) indicates a sequence of random variables \( (X_1, X_{i+1}, \ldots, X_j) \).

Consider the state-dependent discrete memoryless relay channel in Fig. 1, which is denoted by \( (X \times \mathcal{X}, p(y,y_1|x,x_1,s), \mathcal{Y}_1 \times \mathcal{Y}, S) \), where \( p(y,y_1|x,x_1,s) \) is a state-dependent probability distribution. \( Y \in \mathcal{Y} \) and \( Y_1 \in \mathcal{Y}_1 \) are receiver and relay outputs, \( X \in \mathcal{X} \) and \( X_1 \in \mathcal{X}_1 \) are source and relay inputs, respectively, which control the channel along with an i.i.d channel state \( S^{n} \), drawn according to a distribution \( P_S \). Three situations are assumed, \( S^{n} \) is available causally 1) at the relay only, 2) at the source only, 3) both at the source and the relay. We assume that the channel is memoryless. In \( n \) channel uses, the source sends a message \( W \) to the destination with the help of the relay. A codebook

Fig. 1. Relay channel with causal CSI known: to the relay only(A-D), to the source only(B-C), and to both the source and the relay (A-C).

\( p_{X,Y}(x,y) = p_X(x)p_Y(y|x) \)

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of size $n$ and rate $R$ for this channel consists of a message set $\mathcal{W} = \{1, \ldots, 2^{nR}\}$, where the message $W$ is uniformly distributed over the set $\mathcal{W}$, a sequence of encoding functions at the source: $\varphi_i : \mathcal{X}_i \times \{S\} \to \mathcal{Y}_i$, a sequence of encoding functions at the relay: $\psi_i : \mathcal{Y}^{n-1}_i \to \mathcal{X}_i$, for $i = 1, 2, \ldots, n$, and a decoding function at the receiver: $\psi^m : \mathcal{Y}^m \to \mathcal{X}$, in which $\{S\}$ means where CSI is known to the related node, the source or the relay or both.

III. MAIN RESULTS

In this section we investigate three different set ups: The channel state is known in a causal manner 1) at the relay only, 2) at the source only and 3) both at the source and the relay node. In the first two cases, CSI is known asymmetrically (only one of the two transmitter could utilize CSI). For all cases, achievable rates are obtained by coding schemes based on combining CF relaying [12, Theorem 6] and Shannon’s strategy at the informed node. The outline of the proof for the first and second cases (Theorems 1 and 2) are presented. The proof of the third case (Theorem 3) is obtained by combining the encoding schemes of the first and second cases, and it is lengthy. Hence, it is omitted for brevity.

A. Causal CSI at the relay only

When CSI is available only at the relay in a causal manner, using Shannon’s strategy and CF relaying, an achievable rate is found in Theorem 1. We believe that in this scenario CF strategy outperforms DF strategy, because in conventional DF strategies, source and relay transmitters must fully or partially cooperate. In order to do that, the source must know the relay input, which allows the source and relay to utilize a joint codebook to transmit cooperative information. When CSI is available only at the relay and is unknown to the source, which is the case in this part, source and relay can not cooperate.

**Theorem 1:** The capacity of the discrete memoryless state dependent relay channel with CSI available causally at the relay only is lower-bounded by:

$$C \geq R_{\text{en}} = \max \{I(X; Y_1, Y | U_1) \}$$

s.t. $I(U_1; Y) \geq I(Y; S| U_1, Y)$

(1)

where the supremum is taken over all joint p.m.f on $S \times U_1 \times X \times Y_1 \times Y \times Y$ of the form

$$p(s, u_1, x, y_1, y, y_1) = p(s)p(x)p(u_1)p(y_1|u_1, s)p(y|y_1, x, x_2)p(y|y_1, u_1, s)$$

(2)

And in fact $x_1 = f_1(u_1, s)$, where $f_1(u_1, s)$ is an arbitrary deterministic function.

**Remark 1:** Note that, in general, the dependence of $y_1$ on $s$, in addition to $y_1, u_1$, should be considered. This issue helps us at first step, to cancel CSI at the source-relay link, because, CSI is available at the relay’s decoder. Secondly it helps the receiver with partial CSIR, where needed. In order to do the second important issue, the relay could compress the CSI (along with the relay’s received signal, $y_1$) and send it to the receiver. As a simple case, if $y_1 = \emptyset$, relay could compress the CSI only and send it to the receiver. So the receiver could utilize this Partial CSI. This is in congruent with the results in [22].

**Remark 2:** It is difficult to evaluate the presented bound in special cases, due to the auxiliary random variables defined on the extended alphabet.

**Outline of the Proof:** Our proof is based on the random coding scheme, which combines CF and Shannon’s strategies at the relay where CSI is available. Consider a block Markov encoding scheme with $B$ blocks of transmission, each of $n$ symbols. A sequence of $B-1$ messages, $w_i, i = 1, 2, \ldots, B-1$, each selected independently and uniformly over $\mathcal{W}$ is to be sent over the channel in $nB$ transmissions. Note that as $B \to \infty$, for fixed $n$, the average rate $R(B-1)/B$ is arbitrarily close to $R$.

**Random Coding:** For any joint p.m.f defined in (2):

1. Generate $2^{nR}$ i.i.d $x^n$ sequences at the source, each with probability $p(x^n) = \prod_{j=1}^{n} p(x_j)$. Index them as $x^n(w)$ where $w \in [1, 2^{nR}]$.

2. Generate $2^{nR}$ i.i.d $u_1^n$ sequences at the relay. Index them as $u_1^n(t)$ where $t \in [1, 2^{nR}]$. Also, note that in CF strategy the codewords at the relay and the source are independent.

3. For each $u_1^n(t)$, generate $2^{nR_1}$ i.i.d $y^n$ sequences. Index them as $y^n(z,t)$ where $t \in [1, 2^{nR}]$ and $z \in [1, 2^{nR_1}]$.

Similar to [12] we define:

$$p(y^n|u_1^n) = \sum_{s \times x_1 \times y \times y_1} p(s)p(x)p(y_1|s)$$

for $t \in [1, 2^{nR}]$ bins defined as $B(t)$ where $t \in [1, 2^{nR}]$.

**Encoding (at the beginning of block i):** We assume that channel state in each block is causally known at the relay node.

1. Let $w_i$ be the new message to be sent from the source node in block $i$. So the source transmits i.i.d $x^n(w_i)$ sequence.

2. At the relay assume that:

$$\left( y^n(z_{i-1}, t_{i-1}), y^n(i-1), u_1^n(t_{i-1}), s^n(i-1) \right) \in \mathcal{X}(Y_1, Y_1, U_1, S)$$

(3)

And assume that $z_{i-1} \in B(t_i)$. So the relay generates $(u_1^n(t_i))$ in block $i$.

Note that although the channel state in block $i$ is causally known at the relay node (at time $j$, relay knows the CSI only from time $1$ to $j$), but the relay knows the channel state of the block $(i-1)$ completely, i.e. $s^n(i-1)$. Hence upon receiving $s_j(i)$ (the channel state at time $j$ ($1 \leq j \leq n$) in the block $i$), the relay encoder sends $x_1(y_i(i)) = f_1(u_1, t_1, s_j(i))$.

**Decoding (at the end of block i):**

The receiver at the end of block $i$ will decode $w_{i-1}$. In the following, we illustrate the decoding procedure at the end of block $i$:...
1) The relay finds a unique index \( z \) such that:

\[
(y^n_t(z|t_i), y^n(\hat{t}_i), u^n_t(t_i), s^n(\hat{t})) \in A^n(Y_t, Y_1, U_1, S)
\]

There exists such an index \( z \) with high probability, if \( n \) is sufficiently large and

\[
\hat{R}_1 > I(\hat{Y}_1; Y_1, S|U_1)
\]

2) The receiver finds the unique \( \hat{t}_i \) such that

\[
(u^n_t(\hat{t}_i), y^n(\hat{t})) \in A^n(U_1, Y)
\]

This step can be done with small enough probability of error, i.e. \( \hat{t}_i = t_i \) if \( n \) is sufficiently large and:

\[
R_1 \leq I(U_1; Y)
\]

3) The receiver knows \( t_{i-1} \) of the previous block. So it calculates a set of indices \( z \) such that:

\[
L(y^n(z|z_{i-1})) \triangleq (y^n_t(z|z_{i-1}), u^n_t(t_{i-1}), y(\hat{t})) \in A^n(Y_t, Y_1, Y)
\]

Then the receiver declares that \( \hat{z}_{i-1} \) was sent in block \( i \) if:

\[
\hat{z}_{i-1} \in B_{\hat{t}_i} \cap L(y^n(z|z_{i-1}))
\]

So with arbitrarily high probability \( \hat{z}_{i-1} = z_{i-1} \), if \( n \) is sufficiently large and:

\[
\hat{R}_1 - R_1 < I(Y; \hat{Y}_1|U_1)
\]

4) Finally the receiver uses both \( \hat{g}_1^n(z_{i-1}|t_{i-1}) \) and \( y^n(\hat{t}) \) and declares that \( \hat{w}_{i-1} \) was sent in the block \( i \) if it is a unique message such that:

\[
(x^n(\hat{w}_{i-1}), y^n(z_{i-1}|t_{i-1}), u^n_t(t_{i-1})) \in A^n
\]

So with arbitrarily high probability \( \hat{w}_{i-1} = w_{i-1} \) if \( n \) is sufficiently large and:

\[
R < I(X; Y; \hat{Y}_1|U_1)
\]

Now combining (6), (7) and (10) yields:

\[
I(U_1; Y) \geq I(\hat{Y}_1; Y_1|U_1) - I(\hat{Y}_1; Y|U_1) = I(\hat{Y}_1; Y_1|U_1, Y)
\]

Thus (12) and (13) show that the equation in (1) is achievable.

\[\Box\]

\[\text{B. Causal CSI at the source only}\]

Now, we state an achievable rate for the second case, in which only the source knows the CSI causally. At the source Shannon's strategy and at the relay CF strategy are used.

\textit{Theorem 2:} The capacity of the discrete memoryless state dependent relay channel with CSI available causally at the source only is lower-bounded by:

\[
C \geq \sup_{P_{u^n}} I(U_1; \hat{Y}_1, Y_1|X_1)
\]

\[\text{s.t. } I(X_1; Y) \geq I(\hat{Y}_1; Y_1|X_1, Y)\]

where the supremum is taken over all joint p.m.f on \( S \times U \times X_1 \times \hat{Y}_1 \times Y_1 \times Y_1 \times Y_1 \times Y_1 \) of the form

\[
p(s, u, x, y, \hat{y}_1, y_1, \hat{y}_1, y_1, y_1) =
\]

\[
p(s)p(u)p(x|u, s)p(\hat{y}_1)p(y_1|x, s)p(\hat{y}_1|y_1, x_1)
\]

And in fact \( x = f(u, s) \), where \( f(\cdot) \) is an arbitrary deterministic function.

\textit{Remark:} Note that, in this case \( \hat{y}_1 \) and \( s \), given \( y_1, x_1 \), are independent, because CSI is not available at relay and could not be compressed in the relay.

\textit{Outline of the Proof:} As already mentioned, we combined CF strategy at the relay with Shannon's strategy at the source where CSI is available. Similar to the proof of previous theorem, a block Markov encoding scheme is considered.

\textit{Random Coding:} For any joint p.m.f defined in (15):

1) Generate \( 2^{nR} \) i.i.d \( u^n \) sequences at the source. Index them as \( u^n(w) \) where \( w \in \{1, 2^{nR}\} \).

2) Generate \( 2^{nR_1} \) i.i.d \( x^n \) sequences at the relay each with probability \( p(x^n) = \prod_{j=1}^{n} p(x_j) \). Index them as \( x^n(t) \) for \( t \in \{1, 2^{nR_1}\} \). Also, note that in CF strategy the codewords at the relay and the source are independent.

3) For each \( x^n(t) \), generate \( 2^{nR_1} \) i.i.d \( y^n \) sequences each with probability \( p(y^n|x^n(t)) = \prod_{j=1}^{n} p(y_j|x_{j-1}) \). Index them as \( y^n(z) \) where \( t \in \{1, 2^{nR_1}\} \) and \( z \in \{1, 2^{nR_1}\} \).

\[
p(\hat{y}_1|x_1) = \sum_{z \in \{1, 2^{nR_1}\}} p(x_1, x_1, y_1, x_2, x_1) p(\hat{y}_1|x_1, x_1)
\]

4) Randomly partition the set \( \{1, 2^{nR_1}\} \) in to \( 2^{nR_1} \) bins defined as \( B(t) \) where \( t \in \{1, 2^{nR_1}\} \).

\textit{Encoding (at the beginning of block \( i \)):} We assume that the channel state (\( S^n \)) in each block is known causally at the source node.

1) Let \( w_i \) be the new message to be sent from the source node in block \( i \). The source generates \( u^n(w_i) \) in block \( i \). Hence upon receiving \( s_j(\hat{t}) \) (the channel state at time \( j \) \( (1 \leq j \leq n) \) in the block \( i \)), the source encoder sends \( x_j(\hat{t}) = f_j(u_j, w_i, s_j(\hat{t})) \).

2) At the relay assume that \( (\hat{g}_1^n(z_{i-1}|t_{i-1}), y^n(\hat{t}_i)) \in A^n(\hat{Y}_1, Y_1, X_1) \) and assume that \( z_{i-1} \in B(t_i) \). So the relay transmits \( x^n(t_i) \) in the block \( i \).

\textit{Decoding (at the end of block \( i \)):} The receiver at the end of block \( i \) will decode \( w_{i-1} \).

1) The relay seeks the unique index \( z \) such that:

\[
(y^n_t(z|t_i), y^n(\hat{t}_i), x^n(t_i)) \in A^n(Y_t, Y_1, X_1)
\]

For enough large \( n \), the probability that the relay decoder could find such an index \( z \) is arbitrarily high.

\[
\hat{R}_1 > I(Y_1; Y_1|X_1)
\]

2) The receiver finds the unique \( \hat{t}_i \) such that

\[
(x^n(\hat{t}_i), y^n(\hat{t})) \in A^n(X_1, Y_1)
\]

If \( n \) is sufficiently large and

\[
R_i \leq I(X_1; Y)
\]
with high probability we have: \( \hat{t}_i = t_i \).

3) Knowing \( t_{i-1} \) (from the previous block), the receiver makes a list code of indices \( z \) such that:

\[
L(y^n(i-1)) \triangleq \left\{ (y^n(z),x^n_t(i-l-1),y(i-1)) \mid (y^n(\hat{Y}_1,X_1,Y)) \right\}
\]  

Then the receiver looks for a unique index \( \hat{z}_{i-1} \) which belongs to both list code and the relevant bin, i.e.

\[
\hat{z}_{i-1} \in B_{t_i} \cap L(y^n(i-1))
\]

For being \( \hat{z}_{i-1} = z_{i-1} \), small enough probability of error could be achieved, if \( n \) is sufficiently large and

\[
\hat{R}_1 - R_1 < I(Y;\hat{Y}_1 | X_1)
\]

4) Then using both \( y^n(\hat{w}_{i-1}) \) and \( y^n(\hat{z}_{i-1}) \), the receiver declares that \( u_{i-1} \) was sent in the block \( i - 1 \) if there is a unique index \( \hat{w}_{i-1} \) such that:

\[
(u^n(\hat{w}_{i-1}),y^n(i-1),\hat{y}^n(z_{i-1}|\hat{w}_{i-1}),x^n_t(i-l-1)) \in A^n_t
\]

This last step could be done with small probability of error, so \( u_{i-1} = w_{i-1} \), if \( n \) is sufficiently large and:

\[
R < I(U;Y,\hat{Y}_1 | X_1)
\]

Now (18), (19), and (22) result in:

\[
I(X_1;Y) \geq I(\hat{Y}_1;Y_1 | X_1, Y)
\]

Thus we see from (24) and (25), that the equation in (14) is achievable.

C. Causal CSI at the source and the relay

In this part, we investigate the third setup where CSI is available symmetrically to both the source and relay in a causal manner. So, Shannon's strategy could be used in both transmitters. The next theorem gives a lower bound for this scenario, based on the CF relaying.

**Theorem 3:** The capacity of the discrete memoryless state relay channel with CSI available causally at the source and relay is lower-bounded by:

\[
C \geq R_{\text{source-relay}} = \sup I(U;\hat{Y}_1,Y | U_1)
\]

\[\text{s.t.} \quad I(U_1;Y) \geq I(\hat{Y}_1;Y_1, S | U_1, Y)\]

where the supremum is taken over all joint p.m.f on \( S \times U \times U_1 \times X_1 \times \hat{Y}_1 \times Y_1 \times Y_2 \) of the form

\[
p(s,u,u_1,x,x_1,y_1,y_\hat{t}_1) = p(s)p(u)p(u_1)p(x_1|u_1)p(y_1|y_\hat{t}_1|x_1)p(y_\hat{t}_1)p(y_1|u_1, y, y_\hat{t}_1)
\]

And in fact \( x = f(u_s,s), x_1 = f_1(u_1,s) \), where \( f(\cdot) \) and \( f_1(\cdot) \) are two arbitrary deterministic functions.

The proof is similar to the previous theorems and is obtained by combining them, but it is lengthy. So, it is omitted here.

IV. Conclusion

We proposed a coding scheme for the state-dependent relay channel with causal CSI, based on the combination of CF relaying with Shannon's strategy at the node who knows CSI. We investigated three distinct scenarios in discrete memoryless case. Asymmetric condition includes: perfect CSI is causally known 1) at the relay only, 2) at the source only. Also the symmetric condition is the case, where both the source and the relay know perfect CSI in a causal manner. We established lower bounds on the capacity, via using this coding scheme.

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